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# Bruno Numbers and Non Linearizability of Polynomials of Degree more than two (Problems on complex dynamical systems)

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# Brjuno Numbers and Non Linearizability of Polynomials of Degree more than two

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## Abstract

We consider an irrational number  $\alpha$  which is not a Brjuno number. If there exists a cubic polynomial which has a Siegel point of multiplier  $\exp(2\pi i\alpha)$ , for any  $d \geq 4$  there exists a  $d - 2$  dimensional holomorphic family of  $\mathcal{P}_{\lambda,d}$  of which all elements have a Siegel point of multiplier  $\exp(2\pi i\alpha)$ .

## 1 Introduction

Consider a germ of a holomorphic map  $(\mathbb{C}, 0) \rightarrow (\mathbb{C}, 0)$

$$f(z) = \lambda z + a_2 z^2 + a_3 z^3 + \cdots$$

with multiplier  $\lambda$  at  $z = 0$ . And we consider the case that  $|\lambda| = 1$  but  $\lambda$  is not a root of unity. Thus the multiplier  $\lambda$  can be written as

$$\lambda = e^{2\pi i\alpha} \text{ for an } \alpha \in \mathbb{R} - \mathbb{Q}.$$

The origin is said to be an irrationally indifferent fixed point.

The linearization problem for  $f$  is whether or not there exists a holomorphic local change of coordinate  $z = h(w)$  with  $h(0) = 0$  and  $h'(0) \neq 0$  which conjugates  $f$  to the irrational rotation  $w \mapsto \lambda w$  so that

$$h(\lambda w) = f(h(w))$$

near the origin. We say that an irrationally indifferent fixed point is a Siegel point or a Cremer point according as the local linearization is possible or not (cf. [2]).

For a suitable  $t > 0$ , the map  $z \mapsto \frac{1}{t}f(tz)$  is holomorphic and univalent on the unit disk  $\mathbb{D}$ . So we consider the special case that  $f$  is holomorphic and univalent on the unit disk  $\mathbb{D}$ . Let

$$S := \{f; \text{ holomorphic and univalent map on } \mathbb{D}, f(0) = 0, \text{ and } |f'(0)| = 1\},$$

$$S_\lambda := \{f \in S; f'(0) = \lambda\}.$$

Then we can consider the linearization problem for  $f \in S_\lambda$  at the origin.

**Definition 1.1.** A map  $f \in S_\lambda$  will be called *linearizable at the origin* if there exist a neighborhood  $U_f$  of the origin and a map  $H_f$  which is holomorphic and univalent on  $U_f$ , and satisfies

$$H_f(0) = 0, \quad H'_f(0) = 1, \quad f(H_f(z)) = H_f(\lambda z).$$

In this case  $H_f$  will be called a *linearizing map of  $f$  at the origin*, and the connected component of the Fatou set of  $f$  which contains the origin is called a *Siegel disk of  $f$  at the origin*.

In order to state the results on this problem, we should introduce the following.

**Definition 1.2.** For  $\alpha \in \mathbb{R} - \mathbb{Q}$ , we consider the continued fraction expansion

$$\alpha = a_0 + \frac{1}{a_1 + \frac{1}{a_2 + \cdots}}$$

where  $a_0$  is an integral part of  $\alpha$ , and  $\alpha_1 := \alpha - a_0$ .  $a_1$  is an integral part of  $1/\alpha_1$ , and  $\alpha_2 := 1/\alpha_1 - a_1$ .  $a_2$  is an integral part of  $1/\alpha_2$ . Inductively, we define  $a_n$ . And we define the  $n$ -th approximate fraction

$$\frac{p_n}{q_n} := a_0 + \frac{1}{a_1 + \frac{1}{a_2 + \cdots + \frac{1}{a_{n-1} + \frac{1}{a_n}}}}$$

where  $p_n/q_n$  is an irreducible fraction. An  $\alpha \in \mathbb{R} - \mathbb{Q}$  is called a *Brjuno number* if

$$\sum_{i \geq 0} \frac{\log q_{i+1}}{q_i} < +\infty.$$

We define  $\mathcal{B} := \{\alpha \in \mathbb{R} - \mathbb{Q}; \alpha \text{ is a Brjuno number}\}.$

Next we state the known results about this problem.

**Theorem 1.1 (Brjuno).** *If  $\alpha \in \mathcal{B}$ , all maps  $f \in S_{\exp(2\pi i\alpha)}$  are linearizable at the origin.*

And Yoccoz proved that this result is best possible.

**Theorem 1.2 (Yoccoz [5]).** *If  $\alpha \notin \mathcal{B}$ , there exists a map  $f \in S_{\exp(2\pi i\alpha)}$  which is non linearizable at the origin.*

Furthermore for quadratic polynomials, Yoccoz proved the following.

**Theorem 1.3 (Yoccoz [5]).** *If  $\alpha \notin \mathcal{B}$ ,*

$$P(z) = e^{2\pi i\alpha} z + z^2$$

*is non linearizable at the origin.*

For  $d \geq 2$ , we define

$$\mathcal{P}_{\lambda,d} := \{P(z) = \lambda z + a_2 z^2 + \cdots + a_d z^d; (a_2, \dots, a_d) \in \mathbb{C}^{d-1}\} \cong \mathbb{C}^{d-1}$$

For  $\mathcal{P}_{\lambda,d}$ ,  $d \geq 3$ , Pérez-Marco proved the following.

**Theorem 1.4 (Pérez-Marco [4]).** *Fix  $\lambda = \exp(2\pi i\alpha)$  ( $\alpha \notin \mathcal{B}$ ) and  $d \geq 3$ . There exists an open dense subset of  $\mathcal{P}_{\lambda,d}$  of which all elements are non linearizable at the origin.*

**Theorem 1.5 (Pérez-Marco [4]).** *In the same condition as above, for any  $(a_3, \dots, a_d) \in \mathbb{C}$  ( $a_d \neq 0$ ), There exists an open dense subset  $U$  of  $\mathbb{C}$  such that if  $a_2 \in U$ , then  $z \mapsto \lambda z + a_2 z^2 + a_3 z^3 \cdots + a_d z^d$  is non linearizable at the origin.*

We would like to consider the linearizability of polynomials of degree more than two in Section 2.

## 2 Linearizability of polynomials of degree more than two

For  $\lambda = e^{2\pi i\alpha}$  ( $\alpha \in \mathbb{R} - \mathbb{Q}$ ), and  $A \in \mathbb{C}$ , let  $P_{\lambda,A}$  be a cubic monic polynomial

$$P_{\lambda,A}(z) := \lambda z + Az^2 + z^3.$$

Then  $P_{\lambda}$  has a fixed point of multiplier  $\lambda$  at  $z = 0$ . Conversely, for any cubic polynomial with a fixed point of multiplier  $\lambda$ , there exists some  $A \in \mathbb{C}$  such that it is affine conjugate to  $P_{\lambda,A}$ .

**Theorem 2.1.** Fix  $\lambda = e^{2\pi i\alpha}$  ( $\alpha \notin \mathcal{B}$ ) and  $d \geq 4$ . Suppose there exists  $A \in \mathbb{C}$  such that  $P_{\lambda,A}$  is linerizable at the origin, then the family  $\mathcal{P}_{\lambda,d}$  contains a  $d-2$  dimensional holomorphic family of which all elements are linearizable at the origin.

Now we shall prove this main theorem.

## 2.1 Cubic perturbation of univalent maps

Let  $\lambda = e^{2\pi i\alpha}$  for  $\alpha \in \mathbb{R} - \mathbb{Q}$ , and let  $f$  be an element of  $S_\lambda$ . We define, for  $a \in \overline{\mathbb{D}} - \{0\}$ ,  $A \in \mathbb{C}$  and  $b \in \mathbb{C}$ ,

$$f_{a,A,b}(z) := a^{-1}f(az) + Abz^2 + b^2z^3.$$

By definition, the triplet  $(U', U, f)$  is called a polynomial-like map of degree  $d$  if  $U'$  and  $U$  are simply connected proper subdomains of  $\mathbb{C}$ , and  $U'$  is relatively compact in  $U$ , and  $f : U' \rightarrow U$  is a holomorphic and proper map of degree  $d$ .

**Lemma 2.1.** For  $A \in \mathbb{C}$  and  $b \in \mathbb{C}$ , we define

$$R_{A,b} := \frac{10}{9}|A||b| + \frac{15}{2},$$

$$B_{A,b} := 27R_{A,b} + 3|A||b| + \frac{81}{4} = 33|A||b| + \frac{891}{4},$$

$$W := \{z; |z| < R_{A,b}\} \text{ and}$$

$$W_{f,a,A,b} := \{z; |z| < \frac{1}{3}\} \cap f_{a,A,b}^{-1}(W).$$

For  $f \in S$ ,  $a \in \overline{\mathbb{D}} - \{0\}$ ,  $A \in \mathbb{C}$  and  $|b|^2 > B_{A,b}$ , the triplet  $(W_{f,a,A,b}, W, f_{a,A,b})$  is a polynomial-like map of degree 3.

*Proof.* It is sufficient to prove this in  $a = 1$ . Since  $f$  is univalent in  $\mathbb{D}$ , it follows that  $\frac{|z|}{(1+|z|)^2} \leq |f(z)| \leq \frac{|z|}{(1-|z|)^2}$  for  $z \in \mathbb{D}$ . In particular, if  $|z| = 1/3$ , we have  $3/16 \leq |f(z)| \leq 3/4$ . For  $|z| = 1/3$ , it follows that

$$|b^2z^3| - |Abz^2 + f(z)| \geq \frac{|b|^2}{27} - \frac{|A||b|}{9} - \frac{3}{4} > R_{A,b}.$$

Thus  $f_{1,A,b}(\{z; |z| < 1/3\})$  properly contains the disk  $W$ , so  $f_{1,A,b} : W_{f,1,A,b} \rightarrow W$  is proper and  $W_{f,1,A,b}$  is simply connected by the maximum modulus principle. And for  $|z| = 1/3$  and  $z_1 \in W$ , it follows that

$$|b^2z^3 - z_1| \geq |b^2z^3| - R_{A,b} > |Abz^2 + f(z)| \quad \text{and}$$

$$\sqrt[3]{\left|\frac{z_1}{b^2}\right|} < \sqrt[3]{\frac{R_{A,b}}{|b|^2}} \leq \sqrt[3]{\frac{R_{A,b}}{27R_{A,b}}} = \frac{1}{3}$$

since  $|b|^2 > B_{A,b} > 27R_{A,b}$  by definition. Thus by the theorem of Rouché,  $f_{1,A,b} : W_{f,1,A,b} \rightarrow W$  is a proper map of degree three.

If  $W_{f,1,A,b}$  were not connected, then the number of connected components of  $W_{f,1,A,b}$  would be three or two. First, if it were three, the connected component of  $W_{f,1,A,b}$  containing the origin would be conformally mapped to  $W$  by  $f_{1,A,b}$ . However this would contradict Schwarz lemma because  $|\lambda| = 1$ . Second, if it were two, two cases would occur. If  $f_{1,A,b}$  would conformally map the connected component of  $W_{f,1,A,b}$  containing the origin to  $W$ , we could derive a contradiction by the same argument as above. If not, there would exist the connected component  $W'$  of  $W_{f,1,A,b}$  which would not contain the origin. Then  $f_{1,A,b}$  would conformally map  $W'$  onto  $W$ . So there would exist the only one point  $z_0 \in W'$  such that  $f_{1,A,b}(z_0) = 0$ . We define  $\phi := (f_{1,A,b}|_{W'})^{-1}$ , and  $\psi(z) := \phi(R_{A,b}z)$ . Then  $\psi$  would conformally map  $\mathbb{D}$  to  $W'$ , and  $\psi(0) = z_0$  (See figure 1).

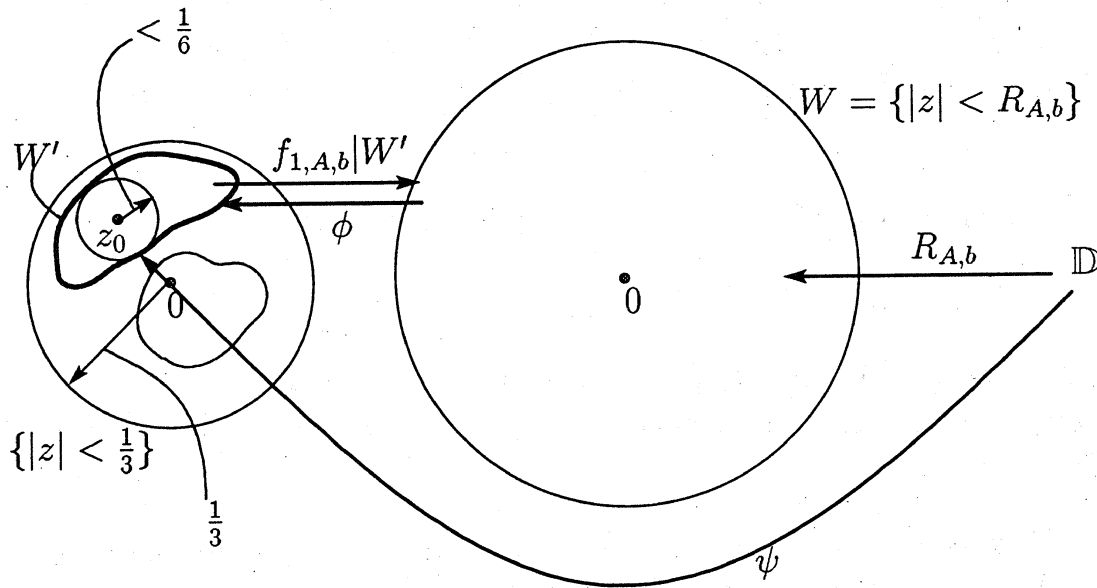


Figure 1:

By the Koebe one-quarter theorem, it follows that  $W'$  contains the open disk of which the radius are  $\frac{1}{4}|\psi'(0)| = \frac{1}{4}R_{A,b}|\phi'(0)|$ . Since  $\mathbb{D}_{1/3} \supset W_{f,1,A,b} \supset W'$  and  $W' \not\ni 0$ , we have  $\frac{1}{4}R_{A,b}|\phi'(0)| < \frac{1}{6}$ . Hence

$$\frac{1}{|\phi'(0)|} > \frac{3R_{A,b}}{2}.$$

On the other hand, we have

$$\frac{1}{|\phi'(0)|} = |f'_{1,A,b}(z_0)| \leq |f'(z_0)| + 2|A||b||z_0| + 3|b|^2|z_0|^2$$

and by the Koebe theorem,  $|f'(z_0)| < 9/2$  for  $|z_0| < 1/3$ . Since  $f_{1,A,b}(z_0) = f(z_0) + Abz_0^2 + b^2z_0^3 = 0$  and  $|f(z_0)| \leq \frac{|z_0|}{(1-|z_0|)^2}$ , we have  $3|b|^2|z_0|^2 + 2|A||b||z_0| < \frac{27}{4} + \frac{5}{3}|A||b|$  for  $|z_0| < 1/3$ . Hence

$$\frac{1}{|\phi'(0)|} < \frac{9}{2} + \frac{27}{4} + \frac{5}{3}|A||b| = \frac{45}{4} + \frac{5}{3}|A||b| = \frac{3}{2}R_{A,b}.$$

That is a contradicton. So  $W_{f,a,A,b}$  is connected, and the proof is completed.  $\square$

**Remark 2.1.**  $|b|^2 > B_{A,b}$  if and only if  $|b| > \frac{33|A| + \sqrt{1089|A| + 891}}{2} =: N(|A|)$ .

## 2.2 Straigtenning of the polynomial-like mapping

Let  $M$  be an arbitrary positive number and take a smooth function  $\eta : \mathbb{R} \rightarrow [0, 1]$  identically 1 on  $(-\infty, 1/3]$  and identically 0 on  $[R_{A,b}, +\infty)$ . And we define the round annulus  $\mathcal{A}(M) := \{b; N(M) < |b| < N(M) + 1\}$ .

For  $f \in S_\lambda$ ,  $a \in \overline{\mathbb{D}} - \{0\}$ ,  $A \in \mathbb{D}_M = \{z; |z| < M\}$  and  $b \in \mathcal{A}(M)$ , we define

$$\tilde{f}_{a,A,b}(z) := \eta(|z|)f_{a,A,b}(z) + (1 - \eta(|z|))(\lambda z + Abz^2 + b^2z^3).$$

Then  $\tilde{f}_{a,A,b} : \mathbb{C} \rightarrow \mathbb{C}$  is  $\mathcal{C}^\infty$  on  $\mathbb{C}$ .

**Lemma 2.2.** *If  $a \rightarrow 0$ , then  $\tilde{f}_{a,A,b}(z)$  converges to  $\lambda z + Abz^2 + b^2z^3$  in  $\mathcal{C}^\infty$ -topology on  $\mathbb{C}$ , and this convergence is uniform in  $f \in S_\lambda$ ,  $A \in \mathbb{D}_M$  and  $b \in \mathcal{A}(M)$ .*

*Proof.* The function  $f_{a,A,b}$  is uniformly convergent to  $\lambda z + Abz^2 + b^2z^3$  on  $\{|z| \leq R_{A,b}\}$  as  $a \rightarrow 0$ . Since  $f(z)$  is univalent on  $\mathbb{D}$ , the coefficients of the power series expansion of it can be estimated uniformly in  $f$ . It is clear that this convergence is unifoum in  $A \in \mathbb{D}_M$  and  $b \in \mathcal{A}(M)$ .  $\square$

We can also prove that two critical points of  $z \mapsto \lambda z + Abz^2 + b^2z^3$  is included in  $\{|z| < 1/3\}$  by the theorem of Rouché. We can conclude the following.

**Lemma 2.3.** *There exists an  $a_0 \in (0, 1]$  and a continuous function  $k : [0, a_0] \rightarrow [0, 1)$  such that  $k(0) = 0$  and for any  $f \in S_\lambda$ ,  $A \in \mathbb{D}_M$  and  $b \in \mathcal{A}(M)$  and  $a \in \overline{\mathbb{D}}_{a_0} - \{0\}$ , the map  $\tilde{f}_{a,A,b}$  is a branched covering map of  $\mathbb{C}$  of degree 3 and it satisfies*

$$\left| \frac{\bar{\partial} \tilde{f}_{a,A,b}(z)}{\partial \tilde{f}_{a,A,b}(z)} \right| \leq k(|a|) \quad (1/3 \leq |z| \leq R_{A,b}).$$

Moreover,  $\frac{\bar{\partial} \tilde{f}_{a,A,b}(z)}{\partial \tilde{f}_{a,A,b}(z)}$  holomorphically depends on  $A \in \mathbb{D}_M$  and  $b \in \mathcal{A}(M)$  and  $a \in \mathbb{D}_{a_0} - \{0\}$ . If  $f$  is a polynomial, this complex dilatation is also holomorphically depends on the coefficients of  $f$ .

For  $f \in S_\lambda$ ,  $A \in \mathbb{D}_M$ ,  $b \in \mathcal{A}(M)$  and  $a \in \mathbb{D}_{a_0} - \{0\}$ , We can define a Beltrami coefficient  $\mu = \mu_{f,a,A,b}$  on  $\mathbb{C}$  such that it is invariant for a pullback of  $\tilde{f}_{a,A,b}$  and it assumes 0 on  $\mathbb{C} - W$  and on  $\bigcap_{n \geq 0} f_{a,A,b}^{-n}(W_{f,a,A,b})$ . Since  $\text{supp } \mu \subset W$  and  $\|\mu\|_\infty \leq k(a) < 1$ , by the Ahlfors-Bers theorem, there exists a unique quasiconformal homeomorphism  $\phi = \phi_{f,a,A,b}$  of  $\mathbb{C}$  onto itself which satisfies the following

- (i) for a.e.  $z \in \mathbb{C}$ ,  $\bar{\partial} \phi(z) = \mu(z) \partial \phi(z)$ ,
- (ii)  $\phi(0) = 0$  and
- (iii)  $\phi(z) - z$  is bounded on  $\mathbb{C}$ .

**Lemma 2.4** (cf. [1]). *There exists an  $A' \in \mathbb{C}$  such that  $\phi \circ \tilde{f}_{a,A,b} \circ \phi^{-1}(z) = \lambda z + A'z^2 + b^2z^3$ , where  $A' \in \mathbb{C}$  holomorphically depends on  $A \in \mathbb{D}_M$ ,  $b \in \mathcal{A}(M)$  and  $a \in \mathbb{D}_{a_0} - \{0\}$ . If  $f$  is a polynomial, it also holomorphically depends on the coefficients of  $f$ .*

*Proof.*  $\phi \circ \tilde{f}_{a,b} \circ \phi^{-1} : \mathbb{C} \rightarrow \mathbb{C}$  is holomorphic, fixes 0 and  $\infty$ . So it is a branched covering map of  $\mathbb{C}$  of degree 3 fixing the origin. Thus we can write

$$\phi \circ \tilde{f}_{a,A,b} \circ \phi^{-1}(z) = \lambda'z + A'z^2 + b'z^3 \quad (\lambda', A', b' \in \mathbb{C}).$$

By the theorem of Naisul ([3]), the multiplier of the fixed point of a holomorphic map is topologically invariant when its module is 1. So we have  $\lambda' = \lambda$ . Next, we would like to show  $b' = b^2$ . According to (iii), we have

$$\phi_{f,a,A,b}(z) = z + c + (\text{lower terms})$$

at a neighborhood of the point at infinity. When  $|z|$  is sufficiently large,  $\tilde{f}_{a,A,b}(z) = \lambda z + Abz^2 + b^2z^3$  by definition, and we note that  $\phi(\tilde{f}_{a,A,b}(z)) = \lambda\phi(z) + A'(\phi(z))^2 + b'(\phi(z))^3$ . Therefore it follows that

$$\begin{aligned} \phi(\lambda z + Abz^2 + b^2z^3) - (\lambda z + Abz^2 + b^2z^3) \\ = (b' - b^2)z^3 + \{(A' - Ab) + 3b'c\}z^2 + (\text{lower terms}). \end{aligned}$$

Since this quantity is bounded as  $|z| \rightarrow +\infty$ , it is necessary that  $b' - b^2 = 0$  and  $A' - Ab + 3b'c = 0$ . Thus it follows that  $b' = b^2$  and  $A' = Ab - 3b^2c$ .  $\square$

**Remark 2.2.** *It is easy to see the following:  $c = c(f, a, A, b)$  holomorphically depends on  $A \in \mathbb{D}_M$ ,  $b \in \mathcal{A}(M)$  and  $a \in \mathbb{D}_{a_0} - \{0\}$ . If  $f$  is a polynomial, it also holomorphically depends on the coefficients of  $f$ . And  $c \rightarrow 0$  uniformly in  $f \in S_\lambda$ ,  $A \in \mathbb{D}_M$  and  $b \in \mathcal{A}(M)$  as  $a \rightarrow 0$ .*



### 3 Completion of proof

Let  $\alpha \notin \mathcal{B}$  and  $\lambda = e^{2\pi i \alpha}$ . Suppose that  $P_{A_0, \lambda}$  is linearizable at the origin. Then for  $b \in \mathbb{C}^*$ ,  $\frac{1}{b}P_{A_0, \lambda}(bz) = \lambda z + A_0 bz^2 + b^2 z^3$  is also linearizable at the origin.

We take  $M = M_0 := 2|A_0| + 1$ . By the Remark 2.2, for any  $\epsilon > 0$  there exists an  $a_1 \in (0, a_0]$  which is independent of  $f \in S_\lambda$ ,  $A \in \mathbb{D}_M$  and  $b \in \mathcal{A}(M_0)$  such that

$$3|b||c(f, a, A, b)| < \epsilon \quad (0 < |a| < a_1).$$

We can take  $\epsilon > 0$  so that  $|A_0| < M_0 - 2\epsilon$ . We define a holomorphic map  $F_{f, a, b}$  on  $\mathbb{D}_{M_0}$ :

$$A \mapsto A - 3bc(f, a, A, b).$$

By the theorem of Rouché, there exists  $A_1 = A_1(f, a, b)$  such that  $F_{f, a, b}(A_1) = A_0$ . We can see that  $A_1 = A_1(f, a, b)$  holomorphically depends on  $b \in \mathcal{A}(M_0)$  and  $a \in \mathbb{D}_{a_1} - \{0\}$ , and if  $f$  is a polynomial, it also holomorphically depends on the coefficients of  $f$ . We can conclude the following.

**Proposition 3.1.** *For any  $f \in S_\lambda$ ,  $b \in \mathcal{A}(M_0)$  and  $a \in \mathbb{D}_{a_1} - \{0\}$ , there exists  $A_1 = A_1(f, a, b)$  which is holomorphic in  $a \in \mathbb{D}_{a_1} - \{0\}$ ,  $b \in \mathcal{A}(M_0)$  and  $f \in S_\lambda$  and also exists  $\phi = \phi_{f, a, A_1, b}$  which is a quasiconformal homeomorphism of  $\hat{\mathbb{C}}$  onto itself which is defined in the previous section such that*

$$\phi \circ \tilde{f}_{a, A_1, b} \circ \phi^{-1}(z) = \frac{1}{b}P_{A_0, \lambda}(bz).$$

*So if there exists  $A_0 \in \mathbb{C}$  such that  $P_{A_0, \lambda}$  is linearizable at the origin,  $f_{a, A_1, b}(z) = a^{-1}f(az) + A_1 bz^2 + b^2 z^3$  is linearizable at the origin.*

In particular, we consider for  $d > 1$ ,

$$\mathcal{U}_d := \{P(z) = \lambda z + a_2 z^2 + \cdots + a_d z^d; \sum_{n=2}^d n|a_n| \leq 1\} \subset S_\lambda.$$

Consequently we can at least conclude the Theorem 2.1.

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